Neutrino Oscillations Induced by Space-Time Torsion

M. Adak[1,2], T. Dereli[2], L. H. Ryder[1]

[1]Department of Physics, University of Kent, Canterbury, Kent CT2 7NF, UK

[2]Department of Physics, Middle East Technical University, 06531 Ankara, Turkey

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Abstract

The gravitational neutrino oscillation problem is studied by considering the Dirac Hamiltonian in a Riemann-Cartan space-time and calculating the dynamical phase. Torsion contributions which depend on the spin direction of the mass eigenstates are found. These effects are of the order of Planck scales.

1 Introduction

Neutrinos are the only particles for which a poem has been written; "Cosmic Gall" by John Updike, quoted in [1]. Recently neutrinos have attracted a lot of attention in high energy physics; see [1, 2] and references therein. A major problem at present is the problem of solar neutrinos. Thermonuclear reactions in the sun produce large number of electron neutrinos; the standard solar model allows us to predict the flux of these neutrinos approximately as 7.3 ± 2.3 SNU[1]. On the other hand experiments which have been done for decades give a measured flux of $2.55 \pm 0.17 \pm 0.18$ SNU – approximately one third of the theoretical rate. This is the solar neutrino problem. A much-publicised solution is "neutrino oscillation (or mixing)". According to neutrino oscillation, in contrast to the conventional view that neutrinos are massless particles, they actually have a mass different from zero, although very small. The essential idea belongs to B.Pontecorvo [3]. Technically speaking it is that the neutrino states of definite mass do not coincide with the weak interaction eigenstates. In other words, an electron neutrino emitted by sun becomes a linear combination of all three neutrinos (electron, muon and tau) while travelling in space towards the earth. Therefore the probability of detecting ν_e emitted by sun as ν_μ or ν_τ on the earth is different from zero. This situation, of course, invokes physics beyond the standard model. Although there are three neutrino families, the essential physics of neutrino oscillations is illustrated very well by considering the interactions of only two of these, which we choose to be ν_e and ν_μ.

All the above arguments have been cast in Minkowski spacetime. However

[1]SNU stands for solar neutrino unit and 1 SNU = 1 capture/sec/10^{36} target atoms

we know that we live in a curved spacetime – perhaps even in a curved spacetime with torsion. Therefore, in more recent years, physicists have turned their attention to specifically gravitational contributions to neutrino oscillations – see [6, 7, 8, 9, 10, 11] and references therein.

In this paper we reinvestigate the effects of gravitation, but we consider a spacetime with torsion, as well as simply curvature; this is the Einstein-Cartan theory, or more specifically the Einstein-Cartan-Dirac theory in our case, since we will be considering Dirac particles in the spacetime. Some work along these lines has already been done [10, 11], but our results do not entirely replicate these findings. The essence of our work is to calculate the dynamical phase of neutrinos, by finding the form of the Hamiltonian, H, from the Dirac equation in Riemann-Cartan spacetime. The phase then follows from the formula

$$i\hbar\frac{\partial\psi}{\partial t} = H\psi \quad \Longrightarrow \quad \psi(t) = e^{-\frac{i}{\hbar}\int H dt}\psi(0) \ . \tag{1}$$

In what follows, ψ is a Dirac spinor and H is a 4×4 matrix; the exponential function is therefore well-defined, as usual, by the series expansion. The Hamiltonian H will depend, for example, on momentum $\vec{p}$, and this is expressed not as a differential operator but simply as a vector.

After we have introduced briefly the Einstein-Cartan-Dirac theory in Sec.2 we find the hamiltonian of a Dirac particle in subsec.2.1. In sec.3 we investigate, in turn, the neutrino oscillations for azimuthal and radial motions. The conclusions are given in sec.4.

2 Einstein-Cartan-Dirac Theory

A Riemann-Cartan space-time is defined by the triple $\{M, g, \nabla\}$ where M is a 4-dimensional differentiable manifold, equipped with a Lorentzian metric g and a metric compatible connection ∇. The metric tensor may be written in terms of orthonormal basis 1-forms $\{e^a\}$ as

$$g = \eta_{ab} e^a \otimes e^b \;\; = \;\; -e^0 \otimes e^0 + e^1 \otimes e^1 + e^2 \otimes e^2 + e^3 \otimes e^3 \ , \tag{2}$$

while the connection is specified by a set of connection 1-forms $\{\Gamma^a{}_b\}$. Metric compatibility requires

$$D\eta_{ab} \equiv d\eta_{ab} - \Gamma^c{}_a \eta_{cb} - \Gamma^c{}_b \eta_{ac} = 0 \ , \tag{3}$$

so that $\Gamma_{ab} = -\Gamma_{ba}$. The Cartan structure equations below define the torsion 2-forms $\{T^a\}$ and the curvature 2-forms $\{R^a{}_b\}$ of the space-time:

$$De^a \;\; \equiv \;\; de^a + \Gamma^a{}_b \wedge e^b = T^a \ , \tag{4}$$

$$D\Gamma^a{}_b \;\; \equiv \;\; d\Gamma^a{}_b + \Gamma^a{}_c \wedge \Gamma^c{}_b = R^a{}_b \ . \tag{5}$$

In this study d, D, $\imath_a$, * denote the exterior derivative, the covariant exterior derivative, the interior derivative and the Hodge star operator, respectively. The local orthonormal frame $\{X_a\}$ is dual to the coframe $\{e^a\}$;

$$\imath_{X_a} e^b \equiv \imath_a e^b = e^b(X_a) = \delta^b_a \ . \tag{6}$$

The space-time orientation is set by the choice $\epsilon_{0123} = +1$. In addition, the Dirac matrices γ^a are in the standard Bjorken and Drell form [12] but for our metric convention they satisfy

$$\gamma^a \gamma^b + \gamma^b \gamma^a = -2\eta^{ab} 1_{4\times 4} \ . \tag{7}$$

We use the formalism of Clifford algebra-valued exterior forms for the Dirac particles. In this formalism $\gamma = \gamma^a e_a$ and $*\gamma = \gamma^a * e_a$. Furthermore, the covariant exterior derivative of spinor fields in the Riemann-Cartan space-time is

$$D\psi = d\psi - \frac{1}{8}\Gamma_{ab}[\gamma^a, \gamma^b]\psi \ . \tag{8}$$

The connection 1-forms, $\{\Gamma^a{}_b\}$, are decomposed as follows

$$\Gamma^a{}_b = \omega^a{}_b + K^a{}_b \ . \tag{9}$$

Here, $\{\omega^a{}_b\}$ are Levi-Civita connection 1-forms which are given by

$$de^a + \omega^a{}_b \wedge e^b = 0 \tag{10}$$

and $\{K^a{}_b\}$ are kontorsion 1-forms which are given by

$$K^a{}_b \wedge e^b = T^a \ . \tag{11}$$

The Einstein-Cartan-Dirac Lagrangian density 4-form is

$$L = -\frac{1}{2\ell^2} R_{ab} \wedge *(e^a \wedge e^b) + \frac{i}{2}\{\overline{\psi} * \gamma \wedge D\psi + \overline{D\psi} \wedge *\gamma\psi\} + \frac{*mc}{\hbar}\overline{\psi}\psi \ . \tag{12}$$

The field equations are then [4] , [5]

$$\begin{aligned} i\hbar * \gamma \wedge (D\psi - \frac{1}{2}T\psi) + *mc\psi &= 0 \ , & (13) \\ R^{bc} \wedge *(e_a \wedge e_b \wedge e_c) &= i\ell^2(\overline{\psi} * \gamma \ \imath_a D\psi - (\imath_a \overline{D\psi}) * \gamma\psi) \ , & (14) \\ T^c \wedge *(e_a \wedge e_b \wedge e_c) &= \frac{\ell^2}{2} e_a \wedge e_b \wedge \overline{\psi}\gamma\gamma_5\psi \ , & (15) \end{aligned}$$

where $T = \imath_a T^a$ is the torsion trace 1-form and $\ell = \sqrt{\frac{Gh}{c^3}} \sim 10^{-31} \ m$ is the Planck length. As is seen from eqn.(15) the source of torsion is spin and magnitude of torsion is proportional to square of the Planck length. In this work, the spinorial contributions to the curvature will be assumed small, and therefore neglected. That is both the Dirac stress-energy tensor on the right hand side of eqn.(14) and the torsion dependent terms on the left hand side are dropped.

The Dirac equation (13) can be recast as

$$i\hbar\gamma^a\imath_a(D\psi - \frac{1}{2}T\psi) - mc\psi = 0\ . \tag{16}$$

Here the torsion trace 1-form T is cancelled by a similar term in the covariant exterior derivative and what is left behind are the totally anti-symmetric axial components of the torsion. These are going to be treated as given background fields.

2.1 Hamiltonian of a Dirac Particle

As mentioned above, our aim is to calculate the phase of a neutrino beam in Riemann-Cartan spacetime. We do this by finding the Hamiltonian for a Dirac particle. The spacetime we consider is Schwarzschild spacetime and the metric is therefore given by

$$g = -f^2 dt\otimes dt + \frac{1}{f^2}dr\otimes dr + r^2(d\theta\otimes d\theta + \sin^2\theta d\varphi\otimes d\varphi)\ . \tag{17}$$

The orthonormal basis coframe is (from (2))

$$e^0 = fcdt \qquad e^1 = \frac{1}{f}dr \qquad e^2 = rd\theta \qquad e^3 = r\sin\theta d\varphi \tag{18}$$

where $f^2 = 1 - \frac{2MG}{rc^2}$. Using (18) and (10) we can calculate the Levi-Civita connection 1-forms, $\omega^a{}_b$. We then substitute eqn.(9) and the Levi-Civita

1-forms into eqn.(8) and then into eqn.(16). Now we use the identity

$$\gamma^a[\gamma^b,\gamma^c] = -2\eta^{ab}\gamma^c + 2\eta^{ac}\gamma^b + 2i\epsilon^{abcd}\gamma_5\gamma_d \tag{19}$$

and eqn.(11), and then write the contorsion 1-forms in component form as $K^a{}_b = K^a{}_{bc}e^c$. The resulting Dirac equation is

$$\begin{aligned}\{\frac{i\hbar}{cf}\gamma^0\frac{\partial}{\partial t} + i\hbar f\gamma^1(\frac{\partial}{\partial r} + \frac{1}{r} + \frac{f'}{2f}) + \frac{i\hbar}{r}\gamma^2(\frac{\partial}{\partial\theta} + \frac{\cot\theta}{2})\\ + \frac{i\hbar}{r\sin\theta}\gamma^3\frac{\partial}{\partial\varphi} - mc + \frac{\hbar}{4}K_{abc}\epsilon^{abcd}\gamma_5\gamma_d\}\psi = 0\end{aligned} \tag{20}$$

where $\gamma_5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$. From the last term we see that it is only the totally anti-symmetric part of the contorsion tensor (in other words, the purely axial component) which contributes to the Dirac equation. [Actually, in theories in which spin is the source of torsion, the contorsion tensor is in any case totally anti-symmetric [13].] Relative to the orthonormal basis 1-forms $\{e^a\}$, we may write

$$K_{abc} = \epsilon_{abcf}A^f \tag{21}$$

for some vector field[2] $A = A^aX_a$. Noting that $\epsilon_{abcf}\epsilon^{abcd} = -3!\delta^d_f$ and defining the components of the momentum operator by

$$\begin{aligned} p_r &= -i\hbar(\frac{\partial}{\partial r} + \frac{1}{r} + \frac{f'}{2f}) & (22)\\ p_\theta &= -\frac{i\hbar}{r}(\frac{\partial}{\partial\theta} + \frac{\cot\theta}{2}) & (23)\\ p_\varphi &= -\frac{i\hbar}{r\sin\theta}\frac{\partial}{\partial\varphi}\ , & (24)\end{aligned}$$

[2]From eqn.(18) $A_0 = \frac{1}{fc}A_t$, $A_1 = fA_r$, $A_2 = \frac{1}{r}A_\theta$, $A_3 = \frac{1}{r\sin\theta}A_\varphi$. This means $\vec{A} = (A_1, A_2, A_3)$ are not the cartesian components, rather the spherical components in the orthonormal basis.

the eqn.(20) becomes

$$
\begin{aligned}
i\hbar\gamma^0\frac{\partial\psi}{\partial t} &= \{f^2\gamma^1 p_r c + f\gamma^2 p_\theta c + f\gamma^3 p_\varphi c \\
&\quad + fmc^2 + \frac{3}{2}\hbar c f\gamma_5\gamma_a A^a\}\psi \ . \qquad (25)
\end{aligned}
$$

Using $\beta \equiv \gamma^0$, $\vec{\alpha} = \beta\vec{\gamma}$ and $\gamma^0(\gamma_5\gamma_a)A^a = \gamma_5 A^0 + \vec{\Sigma}.\vec{A}$ where $A^a = (A^0, \vec{A})$ and $\vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$. Comparing the above with the Schrödinger equation

$$
i\hbar\frac{\partial\psi}{\partial t} = H\psi \qquad (26)
$$

we find that

$$
H = f^2\alpha^1 p_r c + f\alpha^2 p_\theta c + f\alpha^3 p_\varphi c + f\beta mc^2 + \frac{3}{2}\hbar c f A^0\gamma_5 + \frac{3}{2}\hbar c f\vec{\Sigma}.\vec{A} \ . \qquad (27)
$$

3 Neutrino Oscillations

Neutrinos that are produced in the sun arrive on the earth. According to the theory of neutrino oscillations, a neutrino with fixed flavour which is emitted by sun becomes a superposition (in our model) of two neutrinos, i.e., they mix during the journey. In this case, even if only one kind of neutrino were emitted by the sun, the probability of measuring the other kind of neutrino on the earth would not be zero. This is called "neutrino oscillation" and is measured by the interference pattern observed on the earth. The starting point of neutrino oscillation theory is that flavour eigenstates are different from mass eigenstates; each flavour eigenstate is a linear superposition of mass eigenstates;

$$
\begin{aligned}
\nu_e &= \cos\Theta\nu_1 + \sin\Theta\nu_2 \qquad (28) \\
\nu_\mu &= -\sin\Theta\nu_1 + \cos\Theta\nu_2 \qquad (29)
\end{aligned}
$$

where ν_e and ν_μ denotes electron and muon states, respectively, and ν_1 and ν_2 mass eigenstates, and Θ is a mixing angle. It should be clear that this scheme only works if the neutrino masses are different from each other, and therefore in general are non-zero; this means that in general there are right-handed neutrinos as well as left-handed ones. The right-handed neutrinos, however, interact with matter only very weakly. That is why they have not yet been observed.

Now how does the interference work? The wave packets which represent electron and muon neutrinos are constituted by the sum of the wave packets which represent mass eigenstates. These wave packets will travel with different speeds, hence, even though they may be centered at the same point in space at the point of production, there will be a seperation between the packets at the point of detection. In order for interference to be observed, however, the spatial separation between them must not be too large. The waves will then interfere at the detector and we can observe the phase difference. For more detailed discussion see Refs.[9, 14].

The model which explains this phenomenon realistically is the radial motion analysis. We begin the investigation, however, by studying azimuthal motion. Although not realistic, this has the virtue of simplicity; and it has also been considered in Ref.[10], though in a different formalism. Our method of procedure will be to write down the Dirac equation and find phases corresponding to mass eigenstates, then finally calculate phase differences.

3.1 The Azimuthal Motion

The Hamiltonian for the azimuthal motion, that is, $\vec{p} = (p_r, p_\theta, p_\varphi) = (0, 0, p)$ could be written from eqn.(27) as

$$H = f\alpha^3 p_\varphi c + f\beta mc^2 + \frac{3}{2}\hbar c f A^0 \gamma_5 + \frac{3}{2}\hbar c f \vec{\Sigma}.\vec{A}\,. \tag{30}$$

This Hamiltonian couples the positive energy states to the negative energy states because of α^3 and γ_5 :

$$H\psi = E\psi \tag{31}$$

where ψ is a four-component spinor whose first two components correspond to positive energy and second two to negative energy states. Firstly we decouple these states (or block diagonalize the Hamiltonian) by writing

$$\det(\mathrm{H} - \mathrm{E}) = 0 \tag{32}$$

where $E = \begin{pmatrix} E_+ & 0 \\ 0 & E_- \end{pmatrix}$. Therefore the upper block E_+ and lower block E_- satisfy

$$\begin{aligned} E_+ &= fpc + \frac{fm^2c^3}{2p} + \frac{3}{2}\hbar c f(-A^0\sigma^3 + \vec{\sigma}.\vec{A}) && (33) \\ E_- &= -fpc - \frac{fm^2c^3}{2p} + \frac{3}{2}\hbar c f(-A^0\sigma^3 + \vec{\sigma}.\vec{A})\,. && (34) \end{aligned}$$

Here we assume that the torsional terms are small compared with the momentum and mass terms[3]. With a unitary transformation

$$\psi = U\xi \tag{35}$$

[3]Numerical values are given at the end of next subsection

where U is a unitary matrix and $\xi = \begin{pmatrix} \xi_+ \\ \xi_- \end{pmatrix}$, our equation system decouples into two equations

$$
\begin{aligned}
E_+ \xi_+ &= i\hbar \frac{\partial}{\partial t} \xi_+ &(36)\\
E_- \xi_- &= i\hbar \frac{\partial}{\partial t} \xi_- \; . &(37)
\end{aligned}
$$

We will be interested in only the positive energy states for simplicity. Similar calculations may be done for the negative energy states. At the end these states will correspond to the mass eigenstates whose linear superpositions will define, for example, electron and muon neutrinos as given by (28) and (29). Now eqn.(36) can be cast easily as two separate equations by diagonalizing, one for spin up, one for spin down[4];

$$
\begin{aligned}
\{fpc + \frac{fm^2c^3}{2p} + \frac{3}{2}\hbar c f A\}\xi_+^{\uparrow} &= i\hbar \frac{\partial}{\partial t} \xi_+^{\uparrow} &(38)\\
\{fpc + \frac{fm^2c^3}{2p} - \frac{3}{2}\hbar c f A\}\xi_+^{\downarrow} &= i\hbar \frac{\partial}{\partial t} \xi_+^{\downarrow} \; , &(39)
\end{aligned}
$$

whose time evolutions are given by

$$
\begin{aligned}
\xi_+^{\uparrow}(t) &= e^{-i\Phi^{\uparrow}(t)} \xi_+^{\uparrow}(0) &(40)\\
\xi_+^{\downarrow}(t) &= e^{-i\Phi^{\downarrow}}(t) \xi_+^{\downarrow}(0) &(41)
\end{aligned}
$$

where $A = \sqrt{(A^0 - A^3)^2 + (A^1)^2 + (A^2)^2}$. The phase of the spin up state is given by

$$
\Phi^{\uparrow}(t) = \frac{1}{\hbar} \int \{fpc + \frac{fm^2c^3}{2p} + \frac{3}{2}\hbar c f A\} dt \; . \tag{42}
$$

[4]Spin operator is chosen as $\vec{\Sigma}$. Besides, it is only in the MASSLESS limit that the neutrino helicity is ± 1, so the state we are discussing is actually allowed, since we are discussing MASSIVE Dirac neutrinos.

For ultrarelativistic neutrinos $pc \simeq E$ and $cdt \simeq Rd\varphi$, so the above becomes

$$\Phi^{\uparrow} = \frac{1}{\hbar}\{\frac{fER}{c}\Delta\varphi + \frac{fm^2c^3R}{2E}\Delta\varphi + \frac{3}{2}\hbar fAR\Delta\varphi\} \ . \tag{43}$$

Similarly the phase of the spin down state is

$$\Phi^{\downarrow} = \frac{1}{\hbar}\{\frac{fER}{c}\Delta\varphi + \frac{fm^2c^3R}{2E}\Delta\varphi - \frac{3}{2}\hbar fAR\Delta\varphi\} \ . \tag{44}$$

These phases alone do not have an absolute meaning; the quantities relevant for the interference pattern at the observation point are discussed below.

Now consider electron and muon neutrinos given by (28) and (29) above, where ν_1 and ν_2 correspond to ξ_+'s. If electron neutrinos are produced at $t = 0$, the time evolution of ν_e is

$$\begin{aligned} \nu_e(t) &= \cos\Theta\nu_1(t) + \sin\Theta\nu_2(t) \\ &= \cos\Theta e^{-i\Phi_1(t)}\nu_1 + \sin\Theta e^{-i\Phi_2(t)}\nu_2 \ . \end{aligned} \tag{45}$$

The probability of measuring muon neutrino at a later time is given by

$$P(\nu_e \to \nu_\mu) = \sin^2 2\Theta \sin^2 \frac{\Delta\Phi}{2} \tag{46}$$

where $\Delta\Phi = \Phi_2 - \Phi_1$. As is seen from (46) the phase difference has an absolute meaning. Now there are four possibilities.

- Both mass eigenstates are spin up, in which case eqn.(45) becomes

  $$\nu_e(t) = \cos\Theta e^{-i\Phi_1^{\uparrow}(t)}\nu_1 + \sin\Theta e^{-i\Phi_2^{\uparrow}(t)}\nu_2 \ . \tag{47}$$

  From (43) and (44), the phase difference is

  $$\Delta\Phi = \Phi_2^{\uparrow} - \Phi_1^{\uparrow} = \frac{\Delta m^2 c^3}{2\frac{E}{f}\hbar} R\Delta\varphi \ . \tag{48}$$

- Both mass eigenstates are spin down. In this case an analogous equation to (47) holds and the phase difference $\Delta\Phi$ is

$$\Delta\Phi = \Phi_2^{\downarrow} - \Phi_1^{\downarrow} = \frac{\Delta m^2 c^3}{2\frac{E}{f}\hbar} R\Delta\varphi \ . \tag{49}$$

- The first mass eigenstate is spin up, the second spin down, then

$$\Delta\Phi = \Phi_2^{\downarrow} - \Phi_1^{\uparrow} = \{\frac{\Delta m^2 c^3}{2\frac{E}{f}\hbar} - 3fA\} R\Delta\varphi \ . \tag{50}$$

- The first one is spin down, the second spin up, then

$$\Delta\Phi = \Phi_2^{\uparrow} - \Phi_1^{\downarrow} = \{\frac{\Delta m^2 c^3}{2\frac{E}{f}\hbar} + 3fA\} R\Delta\varphi \tag{51}$$

where $\Delta m^2 = m_2^2 - m_1^2$. From (48) and (49) it is seen that if both mass eigenstates have the same spin projection there is no contribution to the oscillation coming from the torsion, but if the mass eigenstates have opposite spin it is seen from (50) and (51) that there is a contribution to the neutrino oscillation coming from the torsion.

When we do an order of magnitude analysis we see that the contribution coming from the torsion is really very small compared with that coming from the mass difference[5]. Here we also emphasise that $\frac{E}{f}$ is the locally measured energy [7]. If there is no torsion, i.e., $A^a = 0$, our result becomes the same as the result of Ref.[10], and in this case there is no gravitational contribution to the phase difference, since the locally measured energy is $\frac{E}{f}$ rather than E. We show below that in the case of radial motion the situation is different.

[5]Numerical values are given at the end of the section 3.2

3.2 The Radial Motion

The Hamiltonian for the radial motion, that is, $\vec{p}=(p,0,0)$ may be written from eqn.(27) as follows

$$H=f^2\alpha^1 p_r c+f\beta mc^2+\frac{3}{2}\hbar cfA^0\gamma_5+\frac{3}{2}\hbar cf\vec{\Sigma}.\vec{A}\,. \tag{52}$$

Applying the same arguments as in the previous subsection we find the upper and lower blocks of the Hamiltonian

$$E_+ \;=\; f^2pc+\frac{m^2c^3}{2p}+\frac{3}{2}\hbar cf(-A^0\sigma^1+\vec{\sigma}.\vec{A}) \tag{53}$$

$$E_- \;=\; -f^2pc-\frac{m^2c^3}{2p}-\frac{3}{2}\hbar cf(-A^0\sigma^1+\vec{\sigma}.\vec{A})\,. \tag{54}$$

As before, we are only interested in the positive energy states which obey

$$E_+\xi_+ \;=\; i\hbar\frac{\partial}{\partial t}\xi_+. \tag{55}$$

Again we first diagonalize eqn.(55) in order to find eigenvalues of spin up and down states. In this case the phases appropriate to the spin up and spin down particles are, respectively,

$$\begin{aligned}\Phi^{\uparrow} &= \frac{1}{\hbar}\{\frac{E}{c}\Delta r-\frac{2MGE}{c^3}\ln\frac{r_B}{r_A}+\frac{m^2c^3}{2E}\Delta r \\ &\quad+\frac{3}{2}\hbar A(\Delta r-\frac{MG}{c^2}\ln\frac{r_B}{r_A})\}\end{aligned} \tag{56}$$

$$\begin{aligned}\Phi^{\downarrow} &= \frac{1}{\hbar}\{\frac{E}{c}\Delta r-\frac{2MGE}{c^3}\ln\frac{r_B}{r_A}+\frac{m^2c^3}{2E}\Delta r \\ &\quad-\frac{3}{2}\hbar A(\Delta r-\frac{MG}{c^2}\ln\frac{r_B}{r_A})\}\end{aligned} \tag{57}$$

where $A=\sqrt{(A^0-A^1)^2+(A^2)^2+(A^3)^2}$. Here we made the assumptions $pc\simeq E$, $cdt\simeq dr$, $f=(1-\frac{2MG}{rc^2})^{1/2}\simeq 1-\frac{MG}{rc^2}$, $\Delta r=r_B-r_A$, where A and

B refer respectively to the points of production and detection; hence r_A is the radius of the sun and r_B is the distance from the center of the sun to the surface of the earth.

If we apply the same procedure as in the previous subsection, from eqn.(56) and (57) there are three different phase differences;

- both mass eigenstates have the same spin state

$$\Delta\Phi = \Phi_2^{\uparrow} - \Phi_1^{\uparrow} = \Phi_2^{\downarrow} - \Phi_1^{\downarrow} = \frac{\Delta m^2 c^3}{2E\hbar}\Delta r \tag{58}$$

- the first is spin up, the second spin down

$$\Delta\Phi = \Phi_2^{\downarrow} - \Phi_1^{\uparrow} = \frac{\Delta m^2 c^3}{2E\hbar}\Delta r - 3A(\Delta r - \frac{MG}{c^2}\ln\frac{r_B}{r_A}) \tag{59}$$

- the first is down, the second up

$$\Delta\Phi = \Phi_2^{\uparrow} - \Phi_1^{\downarrow} = \frac{\Delta m^2 c^3}{2E\hbar}\Delta r + 3A(\Delta r - \frac{MG}{c^2}\ln\frac{r_B}{r_A}) \tag{60}$$

where the mass difference is $\Delta m^2 = m_2^2 - m_1^2$.

If an order of magnitude analysis is done, for vacuum $\Delta m^2 c^4 \sim 10^{-10}\ \ eV^2$, $E \sim 10\ \ \ MeV$, $r_A \equiv R_\odot = 7\times 10^8\ \ \ m$, the earth-sun distance $\Delta r \simeq 1.5\times 10^{11}\ \ m$ and $r_B = \Delta r + r_A \simeq 1.5\times 10^{11}\ \ m$ and for the sun $\frac{MG}{c^2} \sim 1.5\ \ km$. As for the magnitude of the vector A^a, it follows from (15) that

$$A^a \sim \ell^2(\overline{\psi}\psi)\ . \tag{61}$$

Then $\|A^a\| \sim 10^{-62}\ m^{-1}$ and hence the torsional term in (59), (60) is much smaller than the mass difference term. When we put $A^a = 0$ we recover the same result as Ref.[10], but we see that in general there is a contribution to neutrino oscillation coming from the torsion of spacetime.

4 Result

We have treated the gravitational neutrino oscillation problem in a different way from previous authors by calculating the Hamiltonian of a Dirac particle in Riemann-Cartan spacetime and finding the dynamical phase. We began for simplicity by treating the (unrealistic) case of azimuthal motion; this gave us the opportunity to compare with [10], in which azimuthal motion is considered in Riemann spacetime. We have found that there is a torsional contribution which depends on the spin directions of mass eigenstates, eqn.(50) and (51). If we put the torsion equal to zero in these equations we recover the same result as Ref.[10]. Next we studied the radial motion which is a more realistic model for our problem. We found here also a torsional contribution to the neutrino oscillation that is dependent on the spin polarizations of the mass eigenstates. Again, when the torsion vanishes we recover exactly the same result as in ref[10].

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